3.3 The Interaction Term

- Recall from Classical Mechanics (see Goldstein, p. 372) that the Lagrangian density for the electromagnetic field is

$$\mathcal{L} = -\frac{1}{16\pi}\sum_{\mu,\nu}\left(\frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu}\right)^2 - \frac{1}{4\pi}\left(\sum_\mu \frac{\partial A_\mu}{\partial x_\mu}\right)^2$$

$$+ \sum_\mu \frac{j_\mu A_\mu}{c}$$

The last term represents (-) the effective potential energy of interaction between currents and the electromagnetic field.

$$\therefore V(t) = -\frac{1}{c}\int j_\mu A_\mu \, d^3\vec{r}.$$

Since the four-dimensional flux density vector is

$$s_\mu = i\bar{\psi}\gamma_\mu\psi$$

the current is $j_\mu = ie\bar{\psi}\gamma_\mu\psi$.

As in non-relativistic scattering, each vertex contributes a factor of $-iV$. Therefore, we can expect a factor of γ_μ to be associated with each vertex.

- The above is an oversimplification because it ignores the corpuscular nature of the fields - i.e. the fields are not quantized. For example, the quantum nature of the electromagnetic field by expanding A_μ into plane waves according to

$$A_\mu(x) = \frac{1}{\sqrt{\Omega}} \sum_{k,\lambda} \frac{1}{\sqrt{2\omega}} e_\mu^{(\lambda)} \left(c_{k,\lambda} e^{ikx} + c_{k,\lambda}^\dagger e^{-ikx} \right)$$

where $kx = \vec{k}\cdot\vec{r} - \omega t$

Ω = normalizing volume

$e_\mu^{(\lambda)}$ = unit polarization vector.

The field is quantized by interpreting the Fourier amplitudes $c_{k,\lambda}^\dagger$ and $c_{k,\lambda}$ as creation and annihilation operators with commutation relations

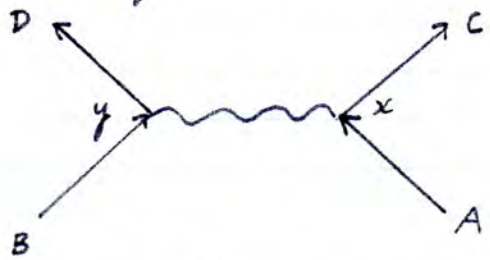
$$[c_{k\lambda}, c_{k'\lambda'}^\dagger] = \delta_{k,k'}\delta_{\lambda,\lambda'}, \quad [c_{k\lambda}, c_{k'\lambda'}] = 0$$

$$[c_{k\lambda}^\dagger, c_{k,\lambda}^\dagger] = 0.$$

This formalism is essential in establishing from first principles the rules for evaluating Feynman diagrams, but needn't be used explicitly once the rules are known.

3.4. The Electron-Electron interaction.

- The diagram to be evaluated is



$$= S_{AB,CD}^{(2)}$$

Multiplying the various pieces together gives

$$S_{AB,CD}^{(2)} = (-i)^2 \int \left[ie\bar{\psi}_D(y)\gamma_\mu \psi_B(y) \right] D^c(y-x) \left[ie\bar{\psi}_c(x)\gamma_\mu \psi_A(x) \right] d^4y\, d^4x$$

in the co-ordinate representation.

Substituting $D^c(y-x) = -\frac{i}{(2\pi)^4}\int d^4k \frac{e^{ik(y-x)}}{k^2 - i\epsilon}$

$$= \frac{-i}{(2\pi)^4}\int_{-\infty}^{\infty} d\omega \int d^3\vec{k}\ \frac{e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2) - i\omega(t_1-t_2)}}{\vec{k}^2 - \omega^2 - i\epsilon}$$

$$\Psi_A(x) = \Psi_A(\vec{r}_1)\, e^{-i\omega_A t_1}$$

$$\bar{\Psi}_c(x) = \bar{\Psi}_c(\vec{r}_1)\, e^{i\omega_c t_1} \quad \text{etc}$$

yields

$$S^{(2)}_{AB,CD} = \frac{-ie^2}{(2\pi)^4}\int \bar{\Psi}_c(\vec{r}_1)\,\gamma_\mu\,\Psi_A(\vec{r}_1)\,d^3\vec{r}_1 \int \bar{\Psi}_D(\vec{r}_2)\,\gamma_\mu\,\Psi_B(\vec{r}_2)\,d^3\vec{r}_2$$

$$\times \int e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}\, d^3\vec{k} \int \frac{d\omega}{\vec{k}^2-\omega^2} \int e^{i(\omega_c-\omega_A-\omega)t_1}\, dt_1$$

$$\times \int e^{i(\omega_D-\omega_B+\omega)t_2}\, dt_2$$

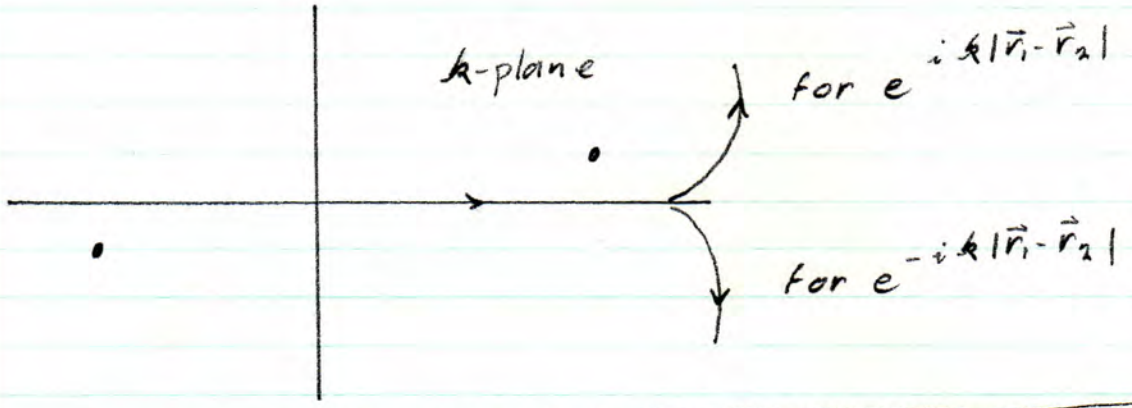
The integration over t_2 gives a factor of $2\pi\delta(\omega_D - \omega_B + \omega)$ so that $\omega = \omega_B - \omega_D$ in the integration over $d\omega$.

- The integral over angles in the photon propagator can also easily be done. For purposes of evaluating the integral, choose the co-ordinate system so that $\vec{r}_1 - \vec{r}_2$ lies in the z-direction. Then

$$\int d^3\vec{k}\ \frac{e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)}}{k^2-\omega^2-i\epsilon} = \int \frac{e^{ik|\vec{r}_1-\vec{r}_2|\cos\theta}}{k^2-\omega^2-i\epsilon}\, k^2 \sin\theta\, dk\, d\theta\, d\varphi$$

$$= \int_0^\infty k^2\, dk\, \frac{2\pi}{k^2-\omega^2-i\epsilon} \times \frac{1}{(ik|\vec{r}_1-\vec{r}_2|)}\left(e^{ik|\vec{r}_1-\vec{r}_2|} - e^{-ik|\vec{r}_1-\vec{r}_2|}\right)$$

$$= \frac{4\pi}{|\vec{r}_1 - \vec{r}_2|} \int_0^\infty \frac{\sin k|\vec{r}_1 - \vec{r}_2|}{k^2 - \omega^2 - i\epsilon} k\, dk$$



$$k^2 - (\omega^2 + i\epsilon) = [k + (\omega^2 + i\epsilon)^{1/2}][k - (\omega^2 + i\epsilon)^{1/2}]$$

$$= \frac{2\pi}{i|\vec{r}_1 - \vec{r}_2|} \frac{(2\pi i)}{2} \left[\frac{e^{+i\omega|\vec{r}_1 - \vec{r}_2|}}{(+2\omega)}(+\omega) + \frac{e^{i\omega|\vec{r}_1 - \vec{r}_2|}}{(-2\omega)}(-\omega) \right]$$

$$= \frac{2\pi^2}{|\vec{r}_1 - \vec{r}_2|} e^{i|\omega||\vec{r}_1 - \vec{r}_2|}$$

Thus $D^c(y-x) = \frac{-i}{2\pi^2} \frac{1}{|\vec{r}_1 - \vec{r}_2|} \int_{-\infty}^{\infty} e^{-i\omega(t_1 - t_2) + i|\omega||\vec{r}_1 - \vec{r}_2|} d\omega$

Substituting into the expression for $S^{(2)}_{AB,CD}$ gives

$$S^{(2)}_{AB,CD} = \frac{i}{4\pi} \int j_{CA}(\vec{r}_1, t)\, j_{BD}(\vec{r}_2, t) \frac{e^{i|\omega_{BD}||\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|}$$

$$d\vec{r}_1\, d\vec{r}_2\, dt$$

where we have substituted
$\omega = \omega_B - \omega_D = \omega_{BD}$

and $j_{CA}(\vec{r}_1, t)$ is a so-called "transition current"

defined by

$$j_{cA}(\vec{r}_1, t) = ie\bar{\psi}_c(x)\gamma\psi_A(x)$$
$$\equiv j_{cA}(\vec{r}_1)\, e^{i\omega_{cA}t}$$

where $j_{cA}(\vec{r}_1) = ie\bar{\psi}_c(\vec{r}_1)\gamma\psi_A(\vec{r}_1)$

Integrating over time gives

$$S^{(2)}_{AB,CD} = \frac{i}{2}\delta(\omega_{cA}+\omega_{BD})\int j_{cA}(\vec{r}_1)\frac{e^{i|\omega_{BD}||\vec{r}_1-\vec{r}_2|}}{|\vec{r}_1-\vec{r}_2|} j_{DB}(\vec{r}_2)$$
$$d^3\vec{r}_1\, d^3\vec{r}_2 .$$

- This can be written in a form that looks like a classical interaction between currents and fields. Assume that $\omega_A > \omega_C$, $\omega_B < \omega_D$ so that $\omega_{AC} > 0$, $\omega_{BD} < 0$.

Then $S^2_{AB,CD} = i\int dt \int j_{DB}(\vec{r}_2, t)\, A_{cA}(\vec{r}_2, t)\, d^3\vec{r}_2$

where $A_{cA}(\vec{r}_2, t) = \frac{1}{4\pi}\int j_{cA}(\vec{r}_1)\frac{e^{-i\omega_{AC}t + i\omega_{AC}|\vec{r}_1-\vec{r}_2|}}{|\vec{r}_1-\vec{r}_2|} d\vec{r}_1$

$$= \frac{1}{4\pi}\int \frac{j_{cA}(\vec{r}_1, t-|\vec{r}_1-\vec{r}_2|)}{|\vec{r}_1-\vec{r}_2|}\, d\vec{r}_1 .$$

which is just the classical expression for the 4-potential produced by the 4-current j_{cA} evaluated at the retarded time $t - |\vec{r}_1 - \vec{r}_2|/c$.

- The above result justifies the "correspondence principle". i.e. write down the classical expression for the

interaction, and then replace currents by transition currents.

- Recall that the interaction energy matrix element $U_{AB,CD}$ is related to $S_{AB,CD}$ by

$$S^{(2)}_{AB,CD} = -2\pi i \delta(\omega_{CA} + \omega_{BD}) U^{(2)}_{AB,CD}.$$

$$\therefore U^{(2)}_{AB,CD} = -\frac{1}{4\pi} \int j_{CA}(\vec{r}_1) j_{DB}(\vec{r}_2) \frac{e^{i|\omega_{AC}||\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|} d^3\vec{r}_1 d^3\vec{r}_2 .$$

3.4. The Low-Energy Approximation

- The product $j_{CA}(\vec{r}_1) j_{DB}(\vec{r}_2)$ is

$$-e^2 \bar{\psi}_C(\vec{r}_1) \bar{\psi}_D(\vec{r}_2) \gamma_\mu(1) \gamma_\mu(2) \psi_A(\vec{r}_1) \psi_B(\vec{r}_2)$$

Remembering that $\vec{\gamma} = -i\beta\vec{\alpha}$, $\gamma_4 = \beta$ and $\bar{\psi} = \psi^* \beta$, we obtain

$$U^{(2)}_{AB,CD} = \frac{e^2}{4\pi} \int \psi_C^*(\vec{r}_1) \psi_D^*(\vec{r}_2) \frac{(1 - \vec{\alpha}_1 \cdot \vec{\alpha}_2)}{|\vec{r}_1 - \vec{r}_2|} \times e^{\frac{i}{c}|\omega_{AC}||\vec{r}_1 - \vec{r}_2|} \psi_A(\vec{r}_1) \psi_B(\vec{r}_2) d^3\vec{r}_1 d^3\vec{r}_2 .$$

where $\left(\frac{e^2}{4\pi}\right)^{1/2}$ = electronic charge.

The exponential factor is called the "retardation factor". Since it depends explicitly on the energies of the wave functions, there is no wave-function-independent operator for which $U^{(2)}_{AB,CD}$

is the matrix element. However, energy independent operators can be found in the form of a power-series expansion in c^{-1} as follows:

$$\text{Expand } \frac{e^{\frac{i}{c}|\omega_{AC}||\vec{r}_1-\vec{r}_2|}}{|\vec{r}_1-\vec{r}_2|} = \frac{1}{|\vec{r}_1-\vec{r}_2|} + \frac{i|\omega_A-\omega_C|}{c} - \frac{(\omega_A-\omega_C)^2}{2c^2}|\vec{r}_1-\vec{r}_2| + \cdots$$

and remember that matrix elements of $\vec{\alpha}_1\cdot\vec{\alpha}_2$ are $O(c^{-2})$.

The matrix element of the second term vanishes by the orthogonality of the wave functions.

The energy dependence is removed from the third term by the commutator trick

$$-(\omega_A-\omega_C)^2|\vec{r}_1-\vec{r}_2| = (\omega_A-\omega_C)(\omega_B-\omega_D)|\vec{r}_1-\vec{r}_2|$$

$$\longrightarrow |\vec{r}_1-\vec{r}_2|H_1H_2 + H_1H_2|\vec{r}_1-\vec{r}_2| - H_1|\vec{r}_1-\vec{r}_2|H_2 - H_2|\vec{r}_1-\vec{r}_2|H_1$$

$$= [H_1,[H_2,|\vec{r}_1-\vec{r}_2|]]$$

with $H = c\vec{\alpha}\cdot\vec{p} + \beta mc^2 + e\hat{A}^{(e)}$. Only the $c\vec{\alpha}\cdot\vec{p}$ term contributes and

$$c^2[\vec{\alpha}_1\cdot\vec{p}_1,[\vec{\alpha}_2\cdot\vec{p}_2,|\vec{r}_1-\vec{r}_2|]] = \frac{\vec{\alpha}_1\cdot\vec{\alpha}_2 - (\vec{\alpha}_1\cdot\hat{n})(\vec{\alpha}_2\cdot\hat{n})}{|\vec{r}_1-\vec{r}_2|^2}, \quad \hat{n} = \frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|}$$

$$\therefore \; -\frac{(\omega_A - \omega_c)^2}{2c^2}|\vec{r}_1 - \vec{r}_2| \longrightarrow \frac{\vec{\alpha}_1 \cdot \vec{\alpha}_2 - (\vec{\alpha}_1 \cdot \hat{n})(\vec{\alpha}_2 \cdot \hat{n})}{2|\vec{r}_1 - \vec{r}_2|}$$

and $(1 - \vec{\alpha}_1 \cdot \vec{\alpha}_2)\dfrac{e^{i/c|\omega_{Ac}||\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|}$

$$\longrightarrow \frac{1}{|\vec{r}_1 - \vec{r}_2|} - \left(\frac{\vec{\alpha}_1 \cdot \vec{\alpha}_2 + (\vec{\alpha}_1 \cdot \hat{n})(\vec{\alpha}_2 \cdot \hat{n})}{2|\vec{r}_1 - \vec{r}_2|} \right)$$

$$= \frac{1}{r_{12}} + B.$$

The second term is called the Breit interaction of order c^{-2}. The first term is the ordinary Coulomb interaction between two electrons.

Question - How can this interaction be used in bound state problems such as the helium atom? We have assumed product type wave functions $\psi_A(\vec{r}_1)\psi_B(\vec{r}_2)$, even though the exact wave function is not separable!

Solution - The exact wave function is expressible as a sum of products of the form

$$\Psi(\vec{r}_1, \vec{r}_2) = \sum_{A,B} c_{A,B}\, \psi_A(\vec{r}_1)\psi_B(\vec{r}_2)$$

with $H_1 \psi_A(\vec{r}_1) = \omega_A \psi_A(\vec{r}_1)$ etc.

Since B has been expressed in a form which does not depend on the one-electron energies, the above result applies to every term in the expression

$$E = \left\langle \sum_{A,B} c_{AB}\psi_A\psi_B \,\middle|\, H_1 + H_2 + \frac{e^2}{r_{12}} + B \,\middle|\, \sum_{A',B'} c_{A',B'}\psi_{A'}\psi_{B'} \right\rangle$$

We can therefore define an effective two-electron Hamiltonian

$$H(1,2) = H_1 + H_2 + \frac{e^2}{r_{12}} + B.$$

Equivalent Non-Relativistic Operator

- Since B is valid only up to terms of $O(c^{-2})$, it is perfectly reasonable to find a non-relativistic operator which yields the same answer to this degree of accuracy. ie find an operator U^{eff} such that

$$\iint \Psi_C^*(\vec{r}_1)\Psi_D^*(\vec{r}_2)\left[\frac{e^2}{r_{12}} + B\right]\Psi_A(\vec{r}_1)\Psi_B(\vec{r}_2)\,d^3\vec{r}_1\,d^3\vec{r}_2$$

$$= \iint \underline{\Phi}_C^*(\vec{r}_1)\underline{\Phi}_D^*(\vec{r}_2)\,U^{eff}\,\underline{\Phi}_A(\vec{r}_1)\underline{\Phi}_B(\vec{r}_2)\,d^3\vec{r}_1\,d^3\vec{r}_2 + O(c^{-4}).$$

where $\underline{\Phi}$ satisfies the ordinary non-relativistic Schrödinger equation. As in the one-electron case, we use the approximate relations

$$\Phi \simeq \left(1 - \frac{p^2}{8m^2c^2}\right)\underline{\Phi}$$

and $\Psi = \begin{pmatrix}\Phi\\ \chi\end{pmatrix}$ with $\chi \simeq \frac{\vec{\sigma}\cdot\vec{p}}{2mc}\Phi$.

Making these substitutions yields a complicated expression, which can be rearranged to obtain U^{eff}. The rearrangement involves a number of integrations by parts. For example, one of the terms coming from e^2/r_{12} is

$$\frac{e^2}{4m^2c^2}\iint[\vec{\sigma}_1\cdot\vec{p}_1\,\varphi_c(\vec{r}_1)]^*\,\varphi_D^*(\vec{r}_2)\,\frac{1}{r_{12}}\,[(\vec{\sigma}_1\cdot\vec{p}_1)\varphi_A(\vec{r}_1)]\varphi_B(\vec{r}_2)$$
$$d^3\vec{r}_1\,d^3\vec{r}_2$$

$$=\frac{e^2}{4m^2c^2}\iint\varphi_c^*(\vec{r}_1)\,\varphi_D^*(\vec{r}_2)\,(\vec{\sigma}_1\cdot\vec{p}_1)\,\frac{1}{r_{12}}\,(\vec{\sigma}_1\cdot\vec{p}_1)\varphi_A(\vec{r}_1)\,\varphi_B(\vec{r}_2)$$
$$d^3\vec{r}_1\,d^3\vec{r}_2\,.$$

Putting all the terms together yields the final result

$$U^{eff}=\frac{e^2}{r_{12}}-\frac{\pi e^2}{m^2c^2}\delta^3(\vec{r}_{12})$$

$$-\frac{e^2}{2m^2c^2}\left[\frac{1}{r_{12}}\,\vec{p}_1\cdot\vec{p}_2+\frac{1}{r_{12}^3}\,\vec{r}_{12}\cdot(\vec{r}_{12}\cdot\vec{p}_1)\,\vec{p}_2\right]\quad\text{orbit-orbit interaction}$$

$$-\frac{e^2}{4m^2c^2r_{12}^3}\Big\{\vec{r}_{12}\times\vec{p}_1\cdot\vec{\sigma}_1-\vec{r}_{12}\times\vec{p}_2\cdot\vec{\sigma}_2$$
$$+2\vec{r}_{12}\times\vec{p}_1\cdot\vec{\sigma}_2-2\vec{r}_{12}\times\vec{p}_2\cdot\vec{\sigma}_1\Big]\quad\text{spin-other-orbit interaction}$$

$$+\frac{e^2}{4m^2c^2}\left[\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{r_{12}^3}-\frac{3(\vec{\sigma}_1\cdot\vec{r}_{12})(\vec{\sigma}_2\cdot\vec{r}_{12})}{r_{12}^5}\right.$$
$$\left.-\frac{8\pi}{3}\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,\delta^3(\vec{r}_{12})\right]\quad\text{spin-spin interaction.}\qquad(37)$$

The first δ-function comes from the replacement $\varphi\to\left(1-\frac{p^2}{8m^2c^2}\right)\Phi$ in

$$\iint\varphi_c^*(\vec{r}_1)\,\varphi_D^*(r_2)\,\frac{1}{r_{12}}\,\varphi_A(\vec{r}_1)\,\varphi_B(\vec{r}_2)\,d^3\vec{r}_1\,d^3\vec{r}_2$$
$$\uparrow$$
$$\left(1-\frac{p_1^2}{8m^2c^2}\right)\Phi_c$$

Integrating by parts gives a term

$$-\frac{p_1^2}{8m^2c^2}\left(\frac{1}{r_{12}}\right) = \frac{\nabla_1^2}{8m^2c^2}\left(\frac{1}{r_{12}}\right) = -\frac{4\pi\delta^3(\vec{r}_{12})}{8m^2c^2}$$

a similar contribution comes from the $\phi_0^*(\vec{r}_2)$ factor to give the final contribution of $-\frac{\pi e^2}{m^2c^2}\delta^3(\vec{r}_{12})$.

- The origin of the second δ-function in the spin-spin interaction is more subtle. This term was missed in early derivations of the Breit interaction. It arises in the integration by parts to obtain the spin-spin interaction as shown. Since high powers of $1/r_{12}$ are involved, the integrated term does not vanish in the limit $|\vec{r}_1 - \vec{r}_2| \to 0$ as one normally expects. The second δ-function takes into account the contributions from the integrated terms. The rest of the derivation is a lengthy, but straightforward extension of the techniques used in the one-electron case.

Final Result

The two-electron terms in (37) can be combined with the one-electron terms in (36)[p.21] to obtain the effective non-relativistic Hamiltonian

$$H = H_0 + \sum_{i=1}^{6} H_i$$

where $H_0 = \frac{1}{2m}\left(p_1^2 + p_2^2\right) + eA_0'$

$$H_1 = -\frac{1}{8m^3c^2}\left[(\vec{\sigma}_1\cdot\vec{\pi}_1)^4 + (\vec{\sigma}_2\cdot\vec{\pi}_2)^4\right]$$

$$H_2 = \frac{-e^2}{2m^2c^2}\left[\frac{1}{r_{12}}\vec{\pi}_1\cdot\vec{\pi}_2 + \frac{1}{r_{12}^3}\vec{r}_{12}\cdot(\vec{r}_{12}\cdot\vec{\pi}_1)\vec{\pi}_2\right]$$

$$H_3 = \frac{\mu}{2mc}\left\{\left[-\vec{E}_1\times\vec{\pi}_1 + \frac{2e}{r_{12}^3}\vec{r}_{12}\times\vec{p}_2\right]\cdot\vec{\sigma}_1 + \left[-\vec{E}_2\times\vec{\pi}_2 + \frac{2e}{r_{12}^3}\vec{r}_{21}\times\vec{p}_1\right]\cdot\vec{\sigma}_2\right.$$

$$H_4 = \frac{-ie\hbar}{4m^2c^2}\left[\vec{p}_1\cdot\vec{E}_1 + \vec{p}_2\cdot\vec{E}_2\right]$$

$$H_5 = \mu^2\left[\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{r_{12}^3} - \frac{3(\vec{\sigma}_1\cdot\vec{r}_{12})(\vec{\sigma}_2\cdot\vec{r}_{12})}{r_{12}^5} - \frac{8\pi}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\,\delta^3(\vec{r}_{12})\right]$$

$$H_6 = \mu\left[\vec{H}_1\cdot\vec{\sigma}_1 + \vec{H}_2\cdot\vec{\sigma}_2\right] + \frac{e}{mc}\left[\vec{A}_1\cdot\vec{p}_1 + \vec{A}_2\cdot\vec{p}_2\right] + \frac{i\mu\hbar}{4mc^2}\left[\frac{\partial\vec{H}_1}{\partial t}\cdot\vec{\sigma}_1 + \frac{\partial\vec{H}_2}{\partial t}\cdot\vec{\sigma}_2\right]$$

and $A_0' = -\frac{Ze}{r_1} - \frac{Ze}{r_2} + \frac{e}{r_{12}}$ + any external potential.

$$\vec{E}_i = -\nabla_i A_0' + \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$$

- Notice that same of the δ-function terms are now contained in H_4.

$$e\nabla_1\cdot\vec{E}_1 = -e\nabla^2 A_0'$$

$$= -4\pi e^2\delta^3(\vec{r}_1) + 4\pi e^2\delta^3(\vec{r}_{12})$$

Application to Helium-Like Atoms

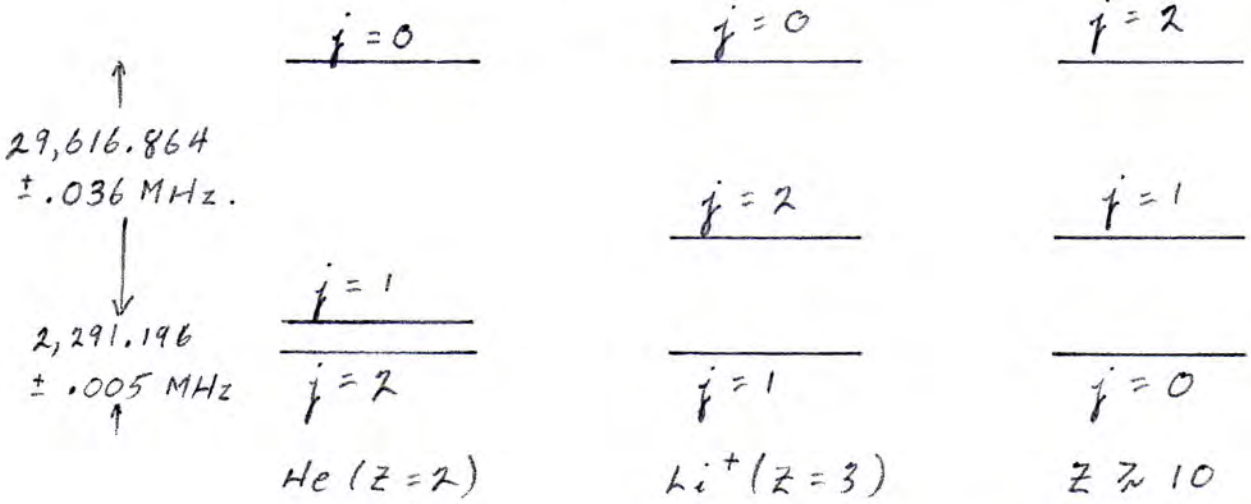
- The most extensively studied state is the fine-structure splitting in the $1s2p\ ^3P_{0,1,2}$ state of helium.

First approximation

- Use hydrogenic wave functions with effective nuclear charge $(Z-1)$ to evaluate the expectation values of H_3 and H_5. H_1, H_2 and H_4 are spin-independent, and so they shift all the fine-structure levels together without changing the splitting. The results are

$$\Delta E(0\to 1) = \frac{\alpha^2 (Z-1)^3}{6n^3}\left[6-(Z-3)\right] \quad Ry.$$

$$\Delta E(1\to 2) = \frac{\alpha^2 (Z-1)^3}{6n^3}\left[-2/5 - 2(Z-2)\right] \quad Ry.$$



The splittings in He can be compared with

$$E(1s2p\ ^1P) - E(1s2p\ ^3P) = 3.07 \times 10^7 \text{ MHz}.$$

Accurate Calculations

- A program was started by C. Schwartz (Phys. Rev. 134, A1181, 1964) to calculate the fine structure splittings to an accuracy of 1 ppm. This is a formidable task because
1) The contribution from B must be taken to second order.
2) Explicit α^4 th. order terms must be evaluated.

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