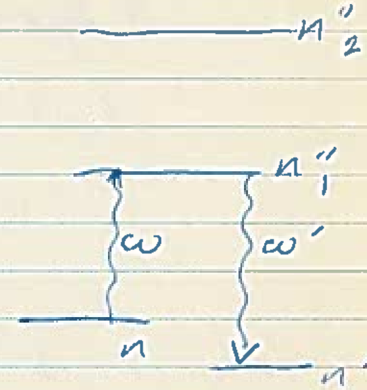
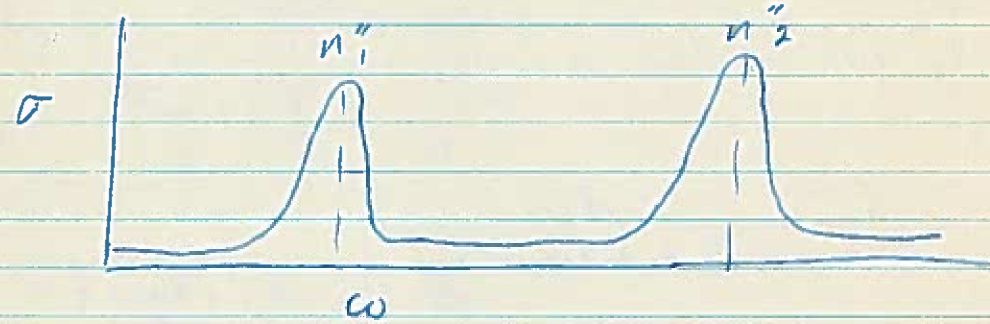
Level Crossing Experiments.



$$\sigma = K \frac{\left|\langle n|\hat{e}_q\cdot\vec{Q}\,|n''\rangle\langle n''|\hat{e}_{q'}\cdot\vec{Q}\,|n'\rangle\right|^2}{(\omega-\omega_{n''n})^2 + \Gamma_{n''}^2/4}$$

for an isolated state n''.



Suppose there are just two intermediate states n_1'', n_2''.

$$\sigma = K \left| \frac{\langle n|\hat{e}_q\cdot\vec{Q}\,|n_1''\rangle\langle n_1''|\hat{e}_{q'}\cdot\vec{Q}\,|n'\rangle}{(\omega-\omega_{n_1''n}) + i\Gamma_{n_1''}} + \frac{\langle n|\hat{e}_q\cdot\vec{Q}\,|n_2''\rangle\langle n_2''|\hat{e}_{q'}\cdot\vec{Q}\,|n'\rangle}{(\omega-\omega_{n_2''n}) + i\Gamma_{n_2''}} \right|^2$$

a typical cross-term is

$$\langle n|\hat{e}_q\cdot\vec{Q}\,|n_1''\rangle\langle n_1''|\,\hat{e}_{q'}\cdot\vec{Q}\,|n'\rangle$$

$$\times\,\langle n'|\,\hat{e}_{q'}\cdot\vec{Q}\,|n_2''\rangle\langle n_2''|\,\hat{e}_q\cdot\vec{Q}\,|n\rangle$$

$$\times\,\frac{1}{[(\omega-\omega_{n_1''n})+i\Gamma_{n_1''}/2][(\omega-\omega_{n_2''n})-i\Gamma_{n_2''}/2]}$$

∴ Suppose that exciting radiation is broad-band so that Lorentz profiles of n_1'' and n_2'' are covered. Total light scattered is found by integrating above over ω.

∴ Frequency factor is

$$\int_0^\infty \frac{d\omega}{\omega^2-\omega\underbrace{\left(\omega_{n_1''n}+\omega_{n_2''n}+i\frac{\Gamma_{n_2''}}{2}-i\frac{\Gamma_{n_1''}}{2}\right)}_{b}+\underbrace{\left(\omega_{n_1''n}-i\frac{\Gamma_{n_1''}}{2}\right)\left(\omega_{n_2''n}+i\frac{\Gamma_{n_2''}}{2}\right)}_{c}}$$

$$=\int_0^\infty\frac{d\omega}{a\omega^2+b\omega+c}$$

$$=\frac{+2}{(4ac-b^2)^{1/2}}\,\mathrm{ctn}^{-1}\left[b/\sqrt{4ac-b^2}\right]$$

$$b^2-4ac=\left(\omega_{n_1''n}-\omega_{n_2''n}-i\frac{\Gamma_{n_2''}}{2}-i\frac{\Gamma_{n_1''}}{2}\right)^2$$

$$=\left\{E_{n_1''}-E_{n_2''}-i\frac{(\Gamma_{n_1''}+\Gamma_{n_2''})}{2}\right\}^2$$

$$=\left(\Delta E_{n_1''n_2''}-i\Gamma_{av.}\right)^2$$

$$\therefore\ (4ac-b^2)^{1/2}=i\Delta E+\Gamma_{av}$$

$$\mathrm{ctn}^{-1}\left(\frac{b}{\sqrt{4ac-b^2}}\right)\to\mathrm{ctn}^{-1}(-\infty)$$

$$=\pi.$$

$$\therefore \sigma = \sum_{n_1'' n_2''} \langle n | \hat{e}_q \cdot \vec{Q} | n_1'' \rangle \langle n_1'' | \hat{e}_{q'} \cdot \vec{Q} | n' \rangle$$

$$\times \langle n' | \hat{e}_{q'} \cdot \vec{Q} | n_2'' \rangle \langle n_2'' | \hat{e}_q \cdot \vec{Q} | n \rangle$$

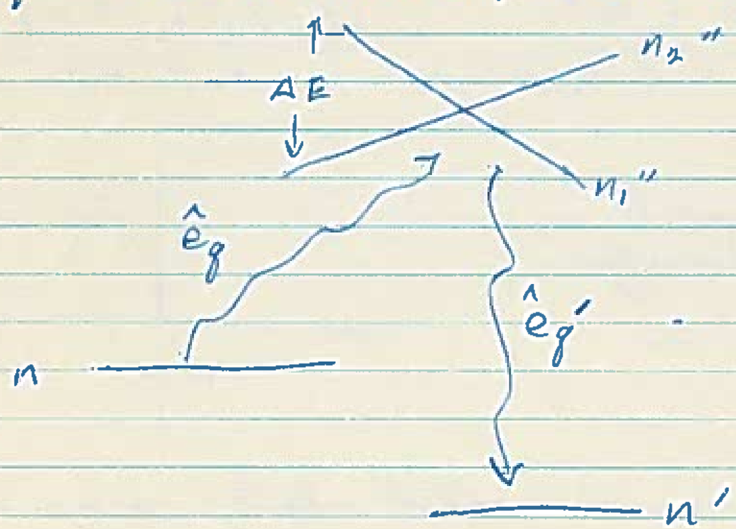
$$\times \frac{1}{i \Delta E_{n_1'' n_2''} + \Gamma_{av}} .$$

$$= \sum_{n_1'' n_2''} \frac{F_{n n_1''} F_{n_2'' n} \, g_{n' n_2''} \, g_{n_1'' n'}}{1 - i \tau \Delta E_{n_2'' n_1''}} .$$

$\tau = 1/\Gamma_{av}$.

$F_{n n_1''} = \langle n | \hat{e}_q \cdot \vec{Q} | n_1'' \rangle$ and $F_{n_2'' n}$ refer to photon absorbed into states n_1'' or n_2''.

$g_{n' n_2''} = \langle n' | \hat{e}_{q'} \cdot \vec{Q} | n_2'' \rangle$ and $g_{n_1'' n'}$ refer to emitted photons.



The sum over n_1'', n_2'' includes $n_1'' = n_2''$, i.e. both "direct" and "cross" terms. If $\tau \Delta E_{n_2'' n_1''} \gg 1$ for all excited states, then only the direct terms contribute and

$$\sigma_0 = \sum_{n_1''} | F_{n n_1''} |^2 | g_{n_1'' n'} |^2 .$$

as discussed previously.

If $\tau \Delta E_{n_1'' n_2''} \stackrel{<}{\sim} 1$ then terms with $n_1'' \neq n_2''$ make a significant contribution.

- Suppose σ is measured as a function of field strength $\mathcal{H}$. In addition to the smoothly varying background signal $\sigma_0(\mathcal{H})$, there is an interference signal in the neighborhood of $E_{n_1''}(\mathcal{H}) = E_{n_2''}(\mathcal{H})$

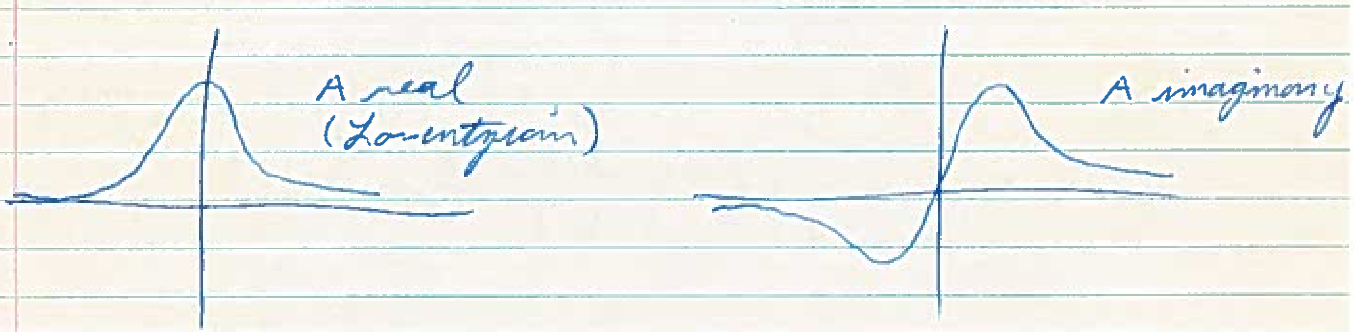
$$\sigma(\mathcal{H}) = \sigma_0(\mathcal{H}) + S.$$

$$S = \frac{A}{1 - i\tau\Delta E_{n_2'' n_1''}} + \frac{A^*}{1 + i\tau\Delta E_{n_2'' n_1''}}$$

$$A = f_{n n_1''} f_{n_2'' n}\, g_{n' n_1''}\, g_{n_2'' n'}$$

$$\text{or } S = \frac{A(1 + i\tau\Delta E)}{1 + \tau^2\Delta E^2} + \frac{A^*(1 - i\tau\Delta E)}{1 + \tau^2\Delta E^2}$$

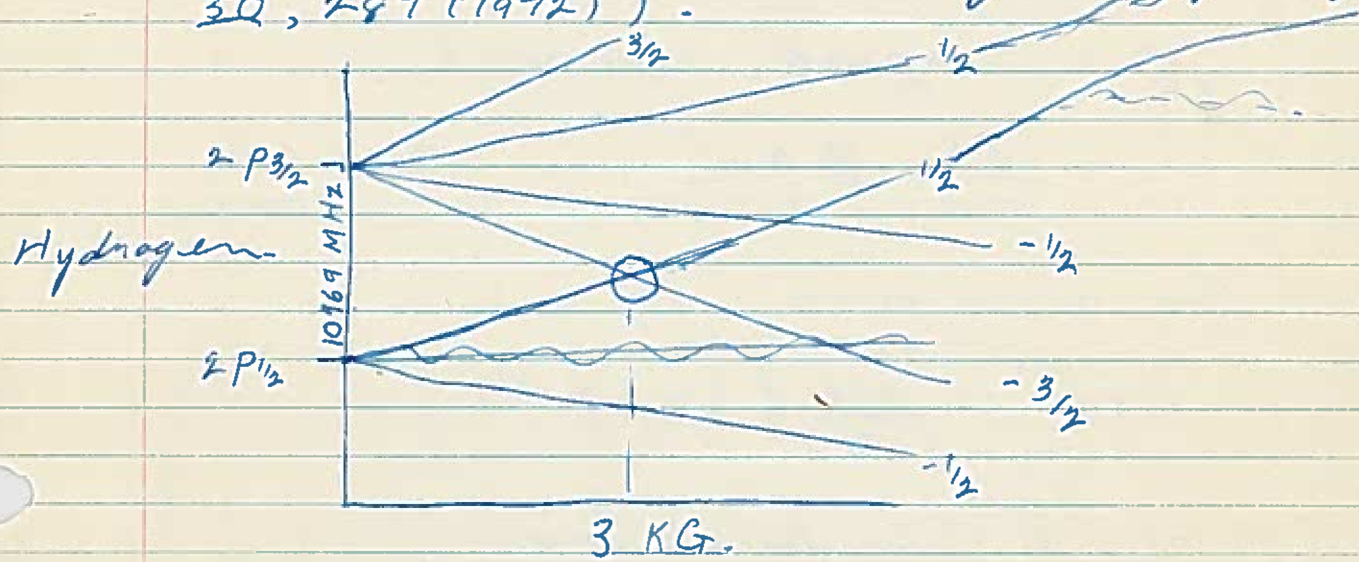
$$= \frac{A + A^*}{1 + \tau^2\Delta E^2} + \frac{(A - A^*)\, i\tau\Delta E}{1 + \tau^2\Delta E^2}$$



Physically, each excited state has a time dependence $e^{-(iE_{n''}/\hbar)t}$. If $E_{n_1''} = E_{n_2''}$, the two states evolve coherently ~~[illegible]~~. If $E_{n_1''} \neq E_{n_2''}$, then coherence is lost in time $\Delta t = \hbar/(E_{n_1''} - E_{n_2''})$. If $\tau < \Delta t$, then photon is emitted before coherence is lost and interference effect appears.

A varies from real → imaginary as relative directions of $\hat{e}_q$ and $\hat{e}_{q'}$ are varied.

- Note that excitation source need not be photons. A low energy electron beam also produces polarized radiation (van der Linde and Dalby, Can. J. Phys. 50, 287 (1972)).



$\Delta m_J = 0, \pm 2$ for simultaneous excitation with linearly polarized radiation.
(Baird et al. P.R. 5, 564, 1972)

used to measure α.

- Like a two-slit diffraction experiment with one photon exciting both states.

Hanle Effect.

- Zero-field level crossing (i.e. m_J degeneracy at zero-field).

$$\Delta E_{m_2'' m_1''} = (m_2'' - m_1'') g_J \mu_B H.$$

$$= 2 g \mu_B H. \quad \text{for } m_2'' - m_1'' = 2.$$

Very small fields are involved.
g_J = Landé g-factor.

The Zeeman Effect

Recall that the magnetic multipole moment operator is

$$Q_{LM}^{(m)} = \nabla(r^L C_{LM}^{*})\left[\frac{e}{mc(L+1)}\underset{\sim}{L} + \underset{\sim}{\mu}\right]$$

$\underset{\sim}{\mu}$ = magnetic moment

$$= \frac{e\hbar}{2mc}\, g_s \underset{\sim}{S}. \qquad \mu_B = \frac{e\hbar}{2mc}$$

For $L=1,\ M=0$

$$Q_{10}^{(m)} = \frac{\mu_B}{\hbar}\left(L_z + g_s S_z\right)$$

$g_s \simeq 2$ (in lowest order)

$$g_s = 2\left(1 + \frac{\alpha}{2\pi} - 0.328\left(\frac{\alpha}{\pi}\right)^2 - \cdots\right)$$

$$= 2.00231923 2$$

For a homogeneous magnetic field,

$\vec{H}$ = const. vector

$$\vec{A} = \frac{1}{2}\vec{H}\times\vec{r} \qquad \nabla\times\vec{A} = \vec{H}.$$

and
$$H_{int} = -\frac{e}{2mc}\left[2\vec{A}\cdot\vec{p} + g_s\vec{S}\cdot\vec{H}\right]$$

$$= -\frac{\mu_B}{\hbar}\left[\vec{L} + g_s\vec{S}\right]\cdot\vec{H}.$$

Take $\vec{H} = H\hat{k}$ as a first-order perturbation. In the presence of the field,

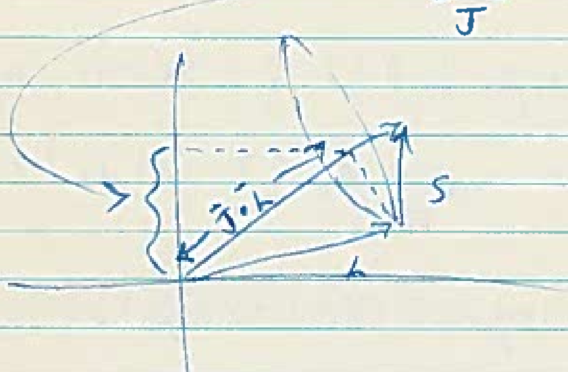
$$E = E_0 + \langle \psi_0 | H_{int} | \psi_0 \rangle + \cdots$$

The diagonal matrix elements ~~are~~ in L, S, J coupling scheme are to be calculated. Recall that

$$\langle J M_J | L_z | J M_J \rangle = \frac{\langle J M_J | J_z (\vec{J}\cdot\vec{L}) | J M_J \rangle}{J(J+1)}$$

$$= \frac{M_J}{J} \frac{\langle J M_J | \vec{J}\cdot\vec{L} | J M_J \rangle}{J+1}$$

Wigner-Eckart Theorem.



$\dfrac{\vec{J}\cdot\vec{L}}{|\vec{J}|}$ = projection of $\vec{L}$ in $\vec{J}$ direction.

$$\vec{J} = \vec{L} + \vec{S}$$

$$\vec{S} = \vec{J} - \vec{L}$$

$$S^2 = J^2 + L^2 - 2\vec{J}\cdot\vec{L}$$

$$\vec{J}\cdot\vec{L} = \frac{J^2 + L^2 - S^2}{2}$$

$$\therefore \langle \alpha L S J M_J | \vec{J}\cdot\vec{L} | \alpha L S J M_J \rangle$$

$$= \frac{J(J+1) + L(L+1) - S(S+1)}{2}$$

Similarly $\langle J M_J | S_z | J M_J \rangle$

$$= \frac{M_J}{J} \frac{\langle J M_J | \vec{J} \cdot \vec{S} | J M_J \rangle}{J+1}$$

$$\vec{J} \cdot \vec{S} = \frac{J^2 + S^2 - L^2}{2}$$

$$\langle J M_J | S_z | J M_J \rangle = \frac{J(J+1) + S(S+1) - L(L+1)}{2}$$

$$\therefore \langle J M_J | H_{int} | J M_J \rangle = -\frac{\mu_B}{\hbar} \langle J M_J | L_z + g_s S_z | J M_J \rangle$$

$$= -\frac{\mu_B}{\hbar} \frac{M_J}{2J(J+1)} \Big\{ J(J+1) + L(L+1) - S(S+1) + g_s \left[J(J+1) + S(S+1) - L(L+1) \right] \Big\}$$

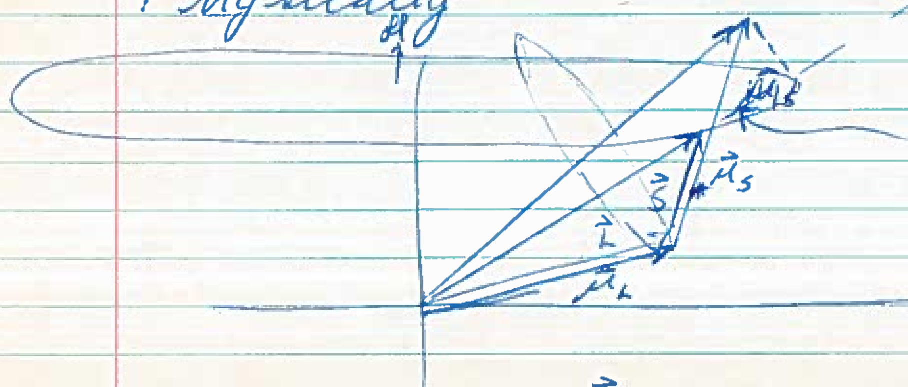
$$= -\frac{\mu_B}{\hbar} g_J M_J = -\frac{\mu_B}{\hbar} g_J \langle J M_J | J_z | J M_J \rangle$$

$$g_J = \frac{1}{2J(J+1)} \Big\{ (1+g_s) J(J+1) + (1-g_s) \left[L(L+1) - S(S+1) \right] \Big\}$$

$$\simeq \frac{3J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

Landé g-factor.

Physically



$\vec{\mu}_L = \mu_B \vec{L}$.

$\vec{\mu}_J = \mu_B g_J$.

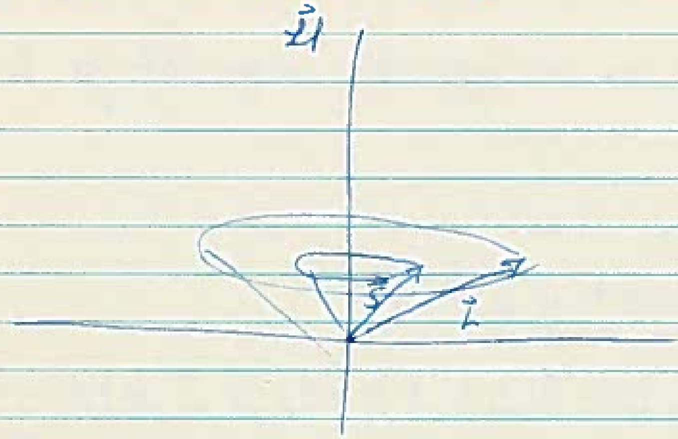
$\vec{\mu}_J$ = effective total magnetic moment resulting from

$$\Delta E = -\vec{\mu}_J \cdot \vec{H}.$$

$$= -\mu_B g_J M_J H.$$

Levels with the same M_J never intersect.

If the field is sufficiently strong, then $\vec{L}$ and $\vec{S}$ precess faster about $\vec{H}$ than about $\vec{J}$.



$$\Delta E = -(\vec{\mu}_L + \vec{\mu}_S)\cdot\vec{H}.$$

J is no longer a good quantum number, only $M_J = M_L + M_S$.

- For intermediate fields, it is necessary to diagonalize explicitly Hint in subset of closely-coupled states.

<u>Inclusion of Hyperfine Structure.</u>

- If field is very small, then fields are of the order of the hyperfine splitting.

$$H = H_a - 2A\,\vec{I}\cdot\vec{J} - \frac{\mu_B}{\hbar}(\vec{L} + g_s\vec{S})\cdot\vec{H}$$

$$- \frac{\mu_N}{\hbar}\, g_I\, \vec{I}\cdot\vec{H}.$$

A = H.F.S. coupling constant.
Classify states according to $\vec{F} = \vec{I} + \vec{J}$,

$\vec{I}$ = nuclear spin.

- Construct eigenfunctions of $\vec{F}$ by vector coupling of product states $|I M_I\rangle|J M_J\rangle$

$$|F M_F\rangle = \sum_{M_I M_J} \langle I J M_I M_J | F M_F \rangle \, | I J M_I M_J \rangle$$

in complete analogy to coupling $\vec{L}$ and $\vec{S}$ to form $\vec{J}$.

as before,

$$F^2 = I^2 + J^2 + 2\vec{I}\cdot\vec{J}$$

$$\therefore \langle I J F M_F | 2\vec{I}\cdot\vec{J} | I J F M_F \rangle = F(F+1) - I(I+1) - J(J+1)$$

$$\langle I J F M_F | I_z | I J F M_F \rangle = \frac{M_F}{F} \langle F M_F | \frac{\vec{F}\cdot\vec{I}}{F+1} | F M_F \rangle$$

$$= \frac{M_F}{2F(F+1)} \left[F(F+1) + I(I+1) - J(J+1) \right]$$

$$\langle F M_F | L_z + g_S S_z | F M_F \rangle = g_J \langle F M_F | J_z | F M_F \rangle$$

$$= g_J \frac{M_F}{F} \langle F M_F | \frac{\vec{F}\cdot\vec{J}}{F+1} | F M_F \rangle$$

$$= g_J \frac{M_F}{2F(F+1)} \left[F(F+1) + J(J+1) - I(I+1) \right]$$

$$\therefore E = E_0^{(J)} + A\left[F(F+1) - I(I+1) - J(J+1) \right]$$

$$- \mu_B g_J \frac{M_F}{2F(F+1)} \left[F(F+1) - I(I+1) + J(J+1) \right] H$$

$$- \mu_N g_I \frac{M_F}{2F(F+1)} \left[F(F+1) + I(I+1) - J(J+1) \right] H.$$

$\mu_N = \dfrac{\mu_B}{1836}$ and can usually be neglected.

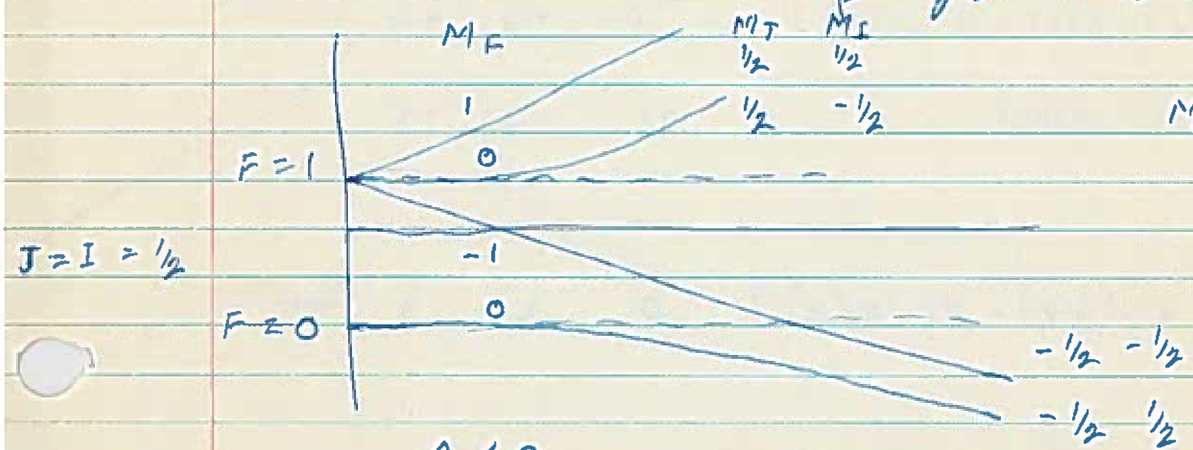
If the field is strong compared to HFS, then $2A\vec{I}\cdot\vec{J}$ is a small perturbation and eigenfunctions are approximately

$$|J M_J\rangle |I M_I\rangle$$

i.e. I and J are uncoupled.

$$\langle \alpha I J M_I M_J | H_{int} | \alpha I J M_I M_J \rangle$$

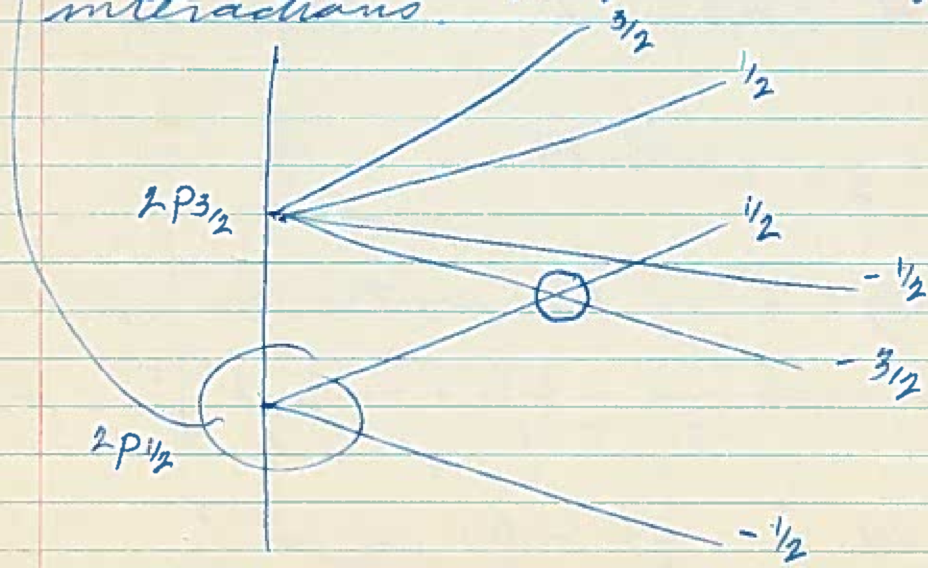
$$\simeq E(\alpha J) - 2A M_I M_J - (\mu_B g_J M_J + \mu_N g_I M_I) \mathcal{H}.$$



$M_F = M_I + M_J$

$A < 0$

Levels with equal M_F ~~and~~ or M_J never cross due to Hyperfine and fine-structure interactions



each line contains two HFS components. $M_F = \pm 1/2$.

The above example corresponds to
$I = J = 1/2$

$F = 1, 0$. as for positronium

$\alpha(1), \beta(1)$ refer to ~~proton~~ electron spin states.
$\alpha(2), \beta(2)$ " " ~~electron~~ proton " "

		M_F	M_I	M_J
$F=1$	$\|11\rangle = \alpha(1)\alpha(2)$	1	1/2	1/2
	$\|10\rangle = \frac{1}{\sqrt{2}}[\alpha(1)\beta(2)+\beta(1)\alpha(2)]$	0	-1/2	+1/2
	$\|1-1\rangle = \beta(1)\beta(2)$	-1	-1/2	-1/2
$F=0$	$\|00\rangle = \frac{1}{\sqrt{2}}[\alpha(1)\beta(2)-\beta(1)\alpha(2)]$	0	1/2	-1/2

Matrix elements are

$$\begin{array}{c|cc} F = & 1 & 0 \\ \hline 1 & & \\ 0 & & \end{array} \quad \begin{pmatrix} +\frac{1}{2}A & \frac{1}{2}\mu_B g_J \mathcal{H} \\ \frac{1}{2}\mu_B g_J \mathcal{H} & -\frac{3}{2}A \end{pmatrix} \qquad M_F = 0$$

$$\frac{1}{2}A + \frac{1}{2}\mu_B g_J M_F \mathcal{H} \qquad M_F = \pm 1$$

Diagonalize as in the positronium problem.

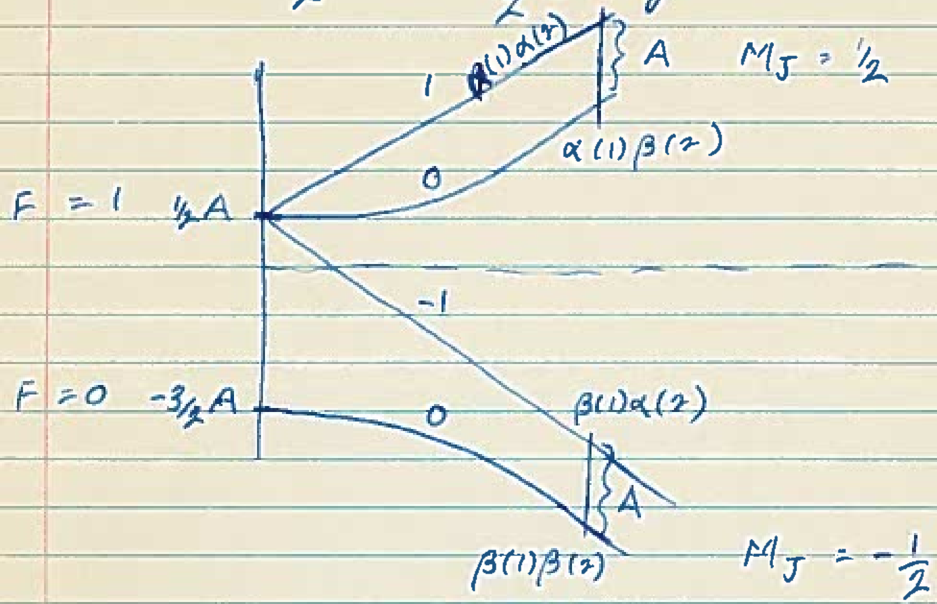
Eigenvalues are $-\frac{1}{2}A \pm A\sqrt{1 + \left(\frac{\mu_B g_J \mathcal{H}}{2A}\right)^2}$ for $M_F = 0$

For $\mathcal{H}$ small, $E = \frac{1}{2}A + \frac{A}{2}\left(\frac{\mu_B g_J \mathcal{H}}{2A}\right)^2$

and $-\frac{3}{2}A - \frac{A}{2}\left(\quad " \quad \right)$

For $\mathcal{H}$ large, $\vec{I}$ and $\vec{J}$ are uncoupled.

$$E = -\frac{1}{2}A \pm \frac{1}{2}\mu_B g_J \mathcal{H}$$



Eigenvectors are determined from

$$\left(1 \mp \sqrt{1+\left(\frac{\beta}{A}\right)^2}\right) a + \frac{\beta}{A} b = 0, \qquad \beta = \frac{1}{2}\mu_B g_J \mathcal{H}.$$

with $|10\rangle' = a\,|10\rangle + b\,|00\rangle$

$|00\rangle' = -b\,|10\rangle + a\,|00\rangle$

For small fields,

$$\left.\begin{aligned} |00\rangle' &= |00\rangle - \frac{\beta}{2A}|10\rangle \\ |10\rangle' &= \frac{\beta}{2A}|00\rangle + |10\rangle \end{aligned}\right\} \text{as for positronium.}$$

As $\beta \to \infty$, $b \to \pm a$ and

$$\left.\begin{aligned} |00\rangle' &\to |00\rangle - |10\rangle = \beta(1)\alpha(2) \\ |10\rangle' &\to |00\rangle + |10\rangle = \alpha(1)\beta(2) \end{aligned}\right\} \vec{I} \text{ and } \vec{J} \text{ are uncoupled.}$$