Application to Photons.

- Each photon state is labelled by wave vector $k$ and polarization $q$ - $\hat{e}_q$ = polarization vector.

The photon vector potential $\vec{A}$ satisfies

$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = 0.$$

Most general solution can be written

$$\vec{A} = \sum_{q} \int \left[ A_{k,q}\, \hat{e}_q\, e^{-i(\vec{k}\cdot\vec{r} - \omega t)} + A^{\dagger}_{k,q}\, \hat{e}_q\, e^{i(\underset{\sim}{k}\cdot\underset{\sim}{r} - \omega t)} \right] d\underset{\sim}{k}$$

with $\omega = c|\vec{k}|$.

Energy of radiation field is

$$H_{Rad} = \frac{1}{8\pi}\int (\mathcal{E}^2 + \mathcal{H}^2)\, d\tau.$$

$$\vec{\mathcal{E}} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t}, \qquad \vec{\mathcal{H}} = \nabla \times \vec{A}.$$

Insert Fourier expansion and use standard vector analysis

$$H_{rad} = \sum_q 4\pi^2 \int k^2 A_{\underset{\sim}{k}q} A_{\underset{\sim}{k}q}^{\dagger}\, d\underset{\sim}{k}$$

- Assume that radiation is in box of sides a, b, c and impose B. C. that $Im(e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}) = 0$ on boundaries. Volume = $\Omega$.

$$k_x = \frac{2\pi n_x}{a}, \quad k_y = \frac{2\pi n_y}{b}, \quad k_z = \frac{2\pi n_z}{c}.$$

~~Not sure of~~.

Divide $k$-space into cells $\frac{2\pi}{a}, \frac{2\pi}{b}, \frac{2\pi}{c}$.

Volume of cell $= (2\pi)^3/abc = \frac{(2\pi)^3}{\Omega} = \Delta$

$abc = \Omega$

~~But $S_k$ = density of states~~

~~$S_k$ = no. of cells between $k$ and $k+dk$~~

$\cancel{= \frac{4\pi k^2 dk}{(2\pi)^3}\Omega = \frac{\Omega k^2 dk}{2\pi^2} \times 2 \text{ polarizations} = \frac{\Omega \omega^2 d\omega}{\pi^2 c^3}}$

Then $\int F(\underset{\sim}{k})\, d\underset{\sim}{k} \longrightarrow \sum_{k} F(\underset{\sim}{k})\, \Delta$

$\therefore\ H_{rad} = 4\pi^2 \sum_{\underset{\sim}{k},q} k^2 A_{\underset{\sim}{k},q} A^\dagger_{\underset{\sim}{k},q}\, \Delta$

In terms of the corresponding $\eta_{\underset{\sim}{k},q}$,

$H_{rad} = \sum_{\underset{\sim}{k},q} \hbar\omega_{\underset{\sim}{k}}\, \eta_{\underset{\sim}{k},q}\, \eta^\dagger_{\underset{\sim}{k},q}$.

Equate, using $\omega = ck$,

$\cancel{4\pi^2 A_{\underset{\sim}{k},q} = \Omega}$

$2\pi\, k\, A_{\underset{\sim}{k},q}\, \Delta^{1/2} = \hbar^{1/2}\omega^{1/2}\, \eta_{\underset{\sim}{k},q}$

$A_{\underset{\sim}{k},q} = \frac{c\hbar^{1/2}}{2\pi\,\omega^{1/2}\,\Delta^{1/2}}\, \eta_{\underset{\sim}{k},q}$.

Then $\vec{A} = \Delta \sum_{k,q} \left\{ A_{\underset{\sim}{k},q}\, \hat{e}_q\, e^{-i(\underset{\sim}{k}\cdot \underset{\sim}{r} - \omega t)} + A^\dagger_{\underset{\sim}{k},q}\, \hat{e}_q\, e^{i(\underset{\sim}{k}\cdot \underset{\sim}{r} - \omega t)} \right\}$

$= \frac{c\hbar^{1/2}}{\omega^{1/2}} \cdot \frac{(2\pi)^{1/2}}{\Omega^{1/2}} \sum_{\underset{\sim}{k},q} \left\{ \eta_{\underset{\sim}{k},q}\, \hat{e}_q\, e^{-i(\underset{\sim}{k}\cdot \underset{\sim}{r} - \omega t)} + \eta^\dagger_{\underset{\sim}{k},q}\, \hat{e}_q\, e^{i(\underset{\sim}{k}\cdot \underset{\sim}{r} - \omega t)} \right\}$

$= c\sqrt{\frac{4\pi}{\Omega}} \sum_{\underset{\sim}{k},q} \frac{1}{\sqrt{2\omega}} \left\{ \qquad '' \qquad \right\}$

Go to p. 105.

## VIII Two-Photon Processes

- Basic processes -

1. Spontaneous two-photon emission.
2. Stimulated " " " .
3. Raman scattering.
4. Two-photon absorption.

Contributions come from two sources.

1) Second-order perturbation.

$$P^{(2)}_{a\to b} = \frac{2\pi}{\hbar^2}\left| \sum_c \frac{\langle b|H|c\rangle\langle c|H|a\rangle}{E_c - E_a}\right|^2 \delta(\omega_{ab})$$

2) Terms involving $A^2$ in 1st. order term

$$P^{(1)}_{a\to b} = \frac{2\pi}{\hbar^2}\left|\langle b|H|a\rangle\right|^2 \delta(\omega_{ab})$$

The general formula is

$P_{n\nu\to n'\nu'}$ (2 photons)

$$= \frac{2\pi}{\hbar^2}\left| \langle n\nu|H_i^{(2)}|n'\nu'\rangle - \sum_{n''\nu''} \frac{\langle n\nu|H_i^{(1)}|n''\nu''\rangle\langle n''\nu''|H_i^{(1)}|n'\nu'\rangle}{\langle n''\nu''|H|n''\nu''\rangle - \langle n\nu|H|n\nu\rangle}\right|$$

$$\times\, \delta(\omega_{nn'} - \omega_{\nu\nu'})$$

$$H_i^{(1)} = \frac{e}{mc}\sum_i \underset{\sim}{A}(\underset{\sim}{r}_i)\cdot \underset{\sim}{p}_i$$

$$H_i^{(2)} = \frac{e^2}{2mc^2}\sum_i \underset{\sim}{A}^2(\underset{\sim}{r}_i)$$

- is diagonal in $n, n'$
to lowest order
i.e. $e^{ik\cdot r} \simeq 1$

Terms involving $H_i^{(1)}$ are

$$\langle n\nu | H_i^{(1)} | n''\nu''\rangle\langle n''\nu'' | H_i^{(1)} | n'\nu'\rangle$$

$\nu$ = no. of photons.

$$= T_1 T_2$$

Consider first two-photon absorption.

$\nu'' = \nu - 1, \quad \nu' = \nu - 2$ if photons are of the same mode.

Since $\langle n\nu | H_i | n'\,\nu-1\rangle = \sqrt{\frac{4\pi}{2\omega\Omega}}\ \langle n | h | n'\rangle \sqrt{\nu_q}$

$$= \sqrt{\frac{4\pi}{2\omega\Omega}}\ i\,\omega_{nn'}\langle n | \hat{e}_q\cdot\vec{Q} | n'\rangle \sqrt{\nu_q}$$

dipole length form.

$$h = \frac{e}{m}\sum_i \hat{e}_q\cdot\vec{p}_i\, e^{i\vec{k}\cdot\vec{r}_i}$$

$$\langle n\nu | H_i^{(1)} | n''\nu''\rangle\langle n''\nu'' | H_i^{(1)} | n'\nu'\rangle$$

$$= \frac{4\pi}{2\omega\Omega}\sqrt{\nu_q(\nu_q-1)}\ \langle n | \hat{e}_q\cdot\vec{Q} | n''\rangle\langle n'' | \hat{e}_q\cdot Q | n'\rangle$$

and $P^{(2)}_{a\to b}$

$$= \frac{8\pi^3}{\hbar^2(\omega\Omega)^2}\,\nu_q(\nu_q-1)\left|\sum_{n''}\frac{\langle n|\hat{e}_q\cdot Q|n''\rangle\langle n''|\hat{e}_q\cdot Q|n'\rangle\,\omega_{nn''}\,\omega_{n''n'}}{\omega-\omega_{nn''}}\right|^2 \delta(\omega_{nn'}-2\omega)$$

$\frac{\nu_q}{\Omega}$ = no. of photons per unit vol.

Agrees with (4-109) of Miyuhima with $4\pi\epsilon_0 = 1$, except for a factor of $2\pi$ which is missing from his equation.

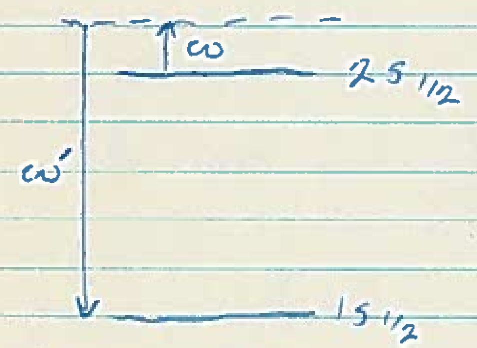
As before $u(\omega) d\omega$

$$u(\omega)d\omega = \frac{N_\omega}{\Omega}(\hbar\omega)d\omega = \text{energy density per unit}$$

$$P^{(2)}_{a\to b} = \frac{8\pi^3}{(\hbar\omega)^4}\left|\sum_{n''} \cdots\right|^2 u(\omega)^2 .$$

## Raman Scattering

- A photon of frequency $\omega$ is absorbed and a second photon of frequency $\omega' = \Delta E + \omega$ is emitted.



The $H_i^{(1)}$ Terms are

$$-\sum_{n'',N''}\frac{\langle n N_\lambda N_{\lambda'}-1|H_i^{(1)}|n''N''\rangle\langle n''N''|H_i^{(1)}|n'N_\lambda-1\, N_{\lambda'}\rangle}{\langle n''N''|H|n''N''\rangle - \langle n N_\lambda N_{\lambda'}-1|H|n N_\lambda N_{\lambda'}-1\rangle}$$

$$= \frac{2\pi}{\Omega^{\#}}\left(\frac{N_\lambda N_\lambda'}{\omega\omega'}\right)^{1/2}\sum_{n''}\left\{\frac{\langle n|\hat{e}_q\cdot\vec{Q}|n''\rangle\langle n''|\hat{e}_{q'}\cdot\vec{Q}|n'\rangle}{\omega_{nn''}+\omega}\right.$$

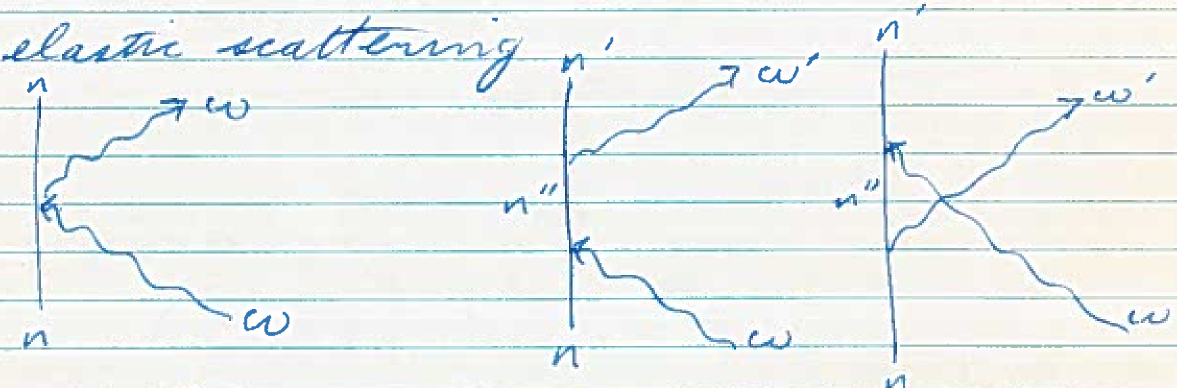
$$\left. + \frac{\langle n|\vec{e}_{q'}\cdot Q|n''\rangle\langle n''|\hat{e}_q\cdot\vec{Q}|n'\rangle}{\omega_{nn''}-\omega'}\right\}\omega_{nn''}\omega_{n''n'}$$

The contribution from $H_i^{(2)}$ is

$$\langle n\, N_\lambda\, N_{\lambda'}-1 \,|\, H_i^{(2)} \,|\, n'\, N_\lambda - 1\, N_{\lambda'} \rangle$$

$$= \frac{2\pi \hbar e^2 Z}{m\, \Omega} \left( \frac{N_\lambda N_{\lambda'}}{\omega \omega'} \right) \delta_{nn'}\, \hat{e}_q \cdot \hat{e}_{q'} .$$

∴ elastic scattering




$H_i^{(2)}$ $\qquad\qquad$ $H_i^{(1)}$

The above two terms can be combined to yield

$$P(n N_\lambda\ N_{\lambda'}-1 \longleftrightarrow n'\ N_\lambda - 1\ N_{\lambda'})$$

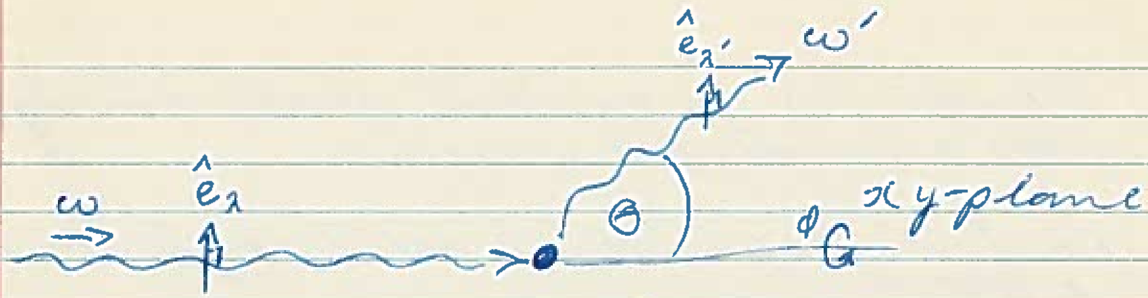
$$= \frac{(2\pi)^3 \omega \omega'}{\hbar^2} \frac{N_\lambda (N_{\lambda'}+1)}{\Omega^2}$$

$$\times \left| \sum_{n''} \frac{\langle n | \hat{e}_\lambda \cdot \vec{Q} | n'' \rangle \langle n'' | \hat{e}_{\lambda'} \cdot \vec{Q} | n' \rangle}{\omega_{nn''} + \omega} + \frac{\langle n | \hat{e}_{\lambda'} \cdot \vec{Q} | n'' \rangle \langle n'' | \hat{e}_\lambda \cdot \vec{Q} | n' \rangle}{\omega_{nn''} - \omega'} \right|^2$$

$$\delta(\omega_{nn'} - \omega + \omega') \qquad\qquad \vec{Q} = e\vec{r}$$

<u>Scattering X-section</u>.

1. Directional correlation.




z-axis $\perp$.

$\hat{e}_\lambda$ and $\hat{e}_{\lambda'}$ are correlated according to $\hat{e}_\lambda \cdot \hat{e}_{\lambda'}$ for s-p-s transitions.

Put $\hat{e}_\lambda = a_\parallel \hat{e}_\parallel + a_\perp \hat{e}_\perp \qquad a_\parallel^2 + a_\perp^2 = 1.$

If $\hat{e}_\lambda = \hat{e}_\perp$, then $\hat{e}_{\lambda'} = \hat{e}_\perp$

and $\hat{e}_\lambda \cdot \hat{e}_{\lambda'} = 1.$

$$\langle 2s | z | 2p_0 \rangle \langle 2p_0 | z | 1s \rangle$$

If $\hat{e}_\lambda = \hat{e}_\parallel$, then $\hat{e}'_\lambda$ also lies in xy-plane and makes an angle $\Theta$ with $\hat{e}_\lambda$. $\hat{e}_\lambda \cdot \hat{e}_{\lambda'} = \cos\Theta$.

$a_\parallel^2$ = prob. of $\parallel$ polarization in incident beam.

$a_\perp^2$ = " " $\perp$ "

$$P(n \leftrightarrow n') = \frac{(2\pi)^3 \omega\omega'}{\hbar^2} \frac{N_\lambda (N_{\lambda'}+1)}{\Omega^2} \left(a_\perp^2 + a_\parallel^2 \cos^2\Theta_A\right)$$

$$\times \Bigg| \underbrace{\sum_{n''} e^2 \langle n | z | n'' \rangle \langle n'' | z | n' \rangle \times \left( \frac{1}{\omega_{nn''} + \omega} + \frac{1}{\omega_{nn''} - \omega'} \right)}_{|Q_{nn'}|^2} \Bigg|^2 \delta(\omega_{nn'} - \omega + \omega')$$

2. Photon $\omega'$ is emitted spontaneously into all available modes.

No. of modes per unit solid angle for fixed polarization

$$\rho(\omega')d\Omega_k = \frac{\omega'^2 d\omega' \, \Omega \; d\Omega_k}{(2\pi)^3 c^3}$$

$$\int \rho(\omega) P(n \to n')^{d\omega' d\Omega_k} = \frac{\omega\omega'^3}{\hbar^2 c^3} \frac{N_\lambda N_{\lambda'}}{\Omega} (a_\perp^2 + a_\parallel^2 \cos^2\theta_k) |Q_{nn'}|_\lambda^2 \, d\Omega_k$$

$\frac{N_\lambda}{\Omega}$ = no. of incident photons / unit vol.

$\frac{c N_\lambda}{\Omega}$ = no. of incident photons crossing unit area / unit time.

$$\sigma \, d\Omega_k = \frac{\text{no. of photons scattered into } d\Omega_k}{cN_\lambda / \Omega}$$

$$= \frac{\omega\omega'^3}{\hbar^2 c^4} (a_\perp^2 + a_\parallel^2 \cos^2\theta_k) |Q_{nn'}|^2 d\Omega_k .$$

<u>Integrated x-section</u>

Let $\varphi$ = angle between observation (scattering) plane and incident polarization plane.

Then $a_\perp = \sin\varphi$
$a_\parallel = \cos\varphi$

Then $\int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta d\theta \, [\sin^2\varphi + \cos^2\varphi \cos^2\theta)$

$$= \pi\left(2 + \frac{2}{3}\right) = \frac{8\pi}{3} .$$

$$\therefore \sigma = \frac{\omega\omega'^3}{\hbar^2 c^4} \frac{8\pi}{3} |Q_{nn'}|^2$$

(dimensions are area as expected)

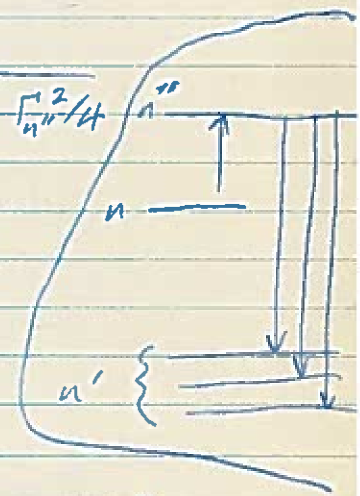
Near a resonance $\left( \omega = -\omega_{nn''} = (E_{n''} - E_n)/\hbar \right)$ one term in $Q_{nn'}$ dominates. As before, replace $E_{n''} \to E_{n''} - i\Gamma_{n''}/2$.

Then

$$\sum_{n'} \int \rho(\omega') P(n \to n') \, d\omega' \, d\Omega_k \quad \text{for } N_{\lambda'} = 1$$

$$= \sum_{n'} \frac{\omega \omega'^3}{c^3} \frac{N_\lambda e_0^4}{\Omega} \frac{8\pi}{3} \frac{|\langle n|z|n''\rangle\langle n''|z|n'\rangle|^2}{(\omega - \omega_{n''n})^2 + \Gamma_{n''}^2/4}$$

$$= \sum_{n'} u(\nu_{nn'}) B_{n\to n''} \cdot \frac{1}{2\pi} \frac{A_{n''\to n'}}{(\omega - \omega_{n''n})^2 + \Gamma_{n''}^2/4}$$




By using $u(\nu)d\nu = \dfrac{(2\pi N_\omega)}{\Omega} h\nu \, d\nu$

$$B_{n\to n''} = \frac{2\pi e^2}{\hbar^2} |\langle n | z | n''\rangle|^2$$

$$A_{n''\to n'} = \frac{4}{3} \frac{\omega'^3 e^2}{c^3 \hbar} \underbrace{|\langle n'' | \vec{r}_i | n'\rangle|^2}_{= |\langle n''|z|n'\rangle|^2}$$

by assumption if $\langle n|z|n''\rangle \neq 0$.

Since $\sum_{n'} A_{n''\to n'} = \Gamma_{n''}$, the above becomes

$$u(\nu_{nn'}) B_{n\to n''} \cdot \underbrace{\frac{1}{\pi} \frac{\Gamma_{n''}/2}{(\omega - \omega_{n''n})^2 + \Gamma_{n''}^2/4}}_{}$$

Lorentz absorption profile for $n \to n''$ transition.

## Stimulated Two-photon Emission

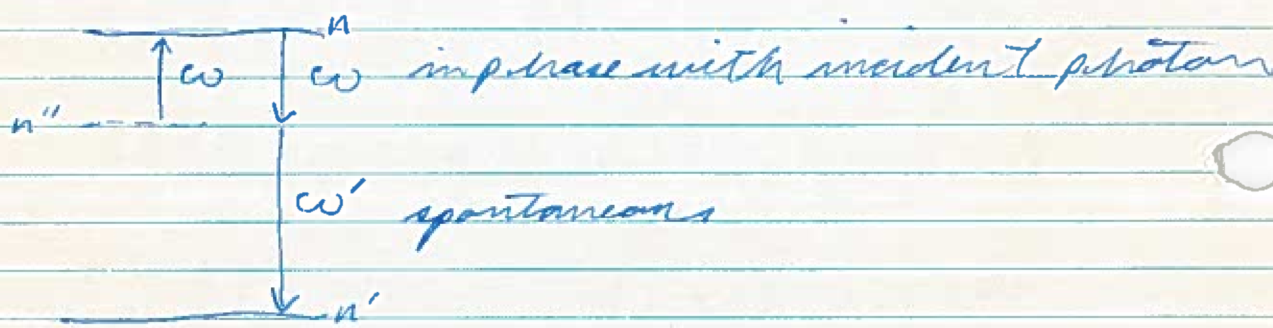
Recall that for Raman scattering,

$$\sigma \, d\Omega_k = \frac{\omega \omega'^3}{c^4} \left( a_\perp^2 + a_\parallel^2 \cos^2\theta_k \right) |Q_{nn'}|^2 d\Omega_k$$

$$Q_{nn'} = \sum_{n''} \langle n|z|n''\rangle\langle n''|z|n'\rangle$$

$$\times \left( \frac{1}{\omega_{nn''} + \omega} + \frac{1}{\omega_{nn''} - \omega'} \right)$$

For stimulated two-photon emission,




$$\hbar \omega' = E_n - E_{n'} - \hbar\omega$$

$$\sigma' d\Omega_k = - \quad — \quad — \quad — \quad |Q'_{nn'}|^2 d\Omega_k$$

$$Q_{nn'} = \sum_{n''} \quad - \quad — \quad ——$$

$$\times \left( \frac{1}{\omega_{nn''} - \omega} + \frac{1}{\omega_{nn''} - \omega'} \right)$$

no resonant terms if no intermediate states.

$$\hat{e}\cdot\vec{r} = -e_- r_+ - e_+ r_- + e_0 r_0 \quad , \qquad e_\pm = \mp\frac{1}{\sqrt{2}}(e_x \pm i e_y)$$

Then $\displaystyle\sum_{m_\ell''=-1}^{1} \langle ns|\hat{e}\cdot\vec{r}|n''p, m_\ell''\rangle\langle n''p, m_\ell''|\hat{e}'\cdot\vec{r}|n's\rangle$

$$= \langle ns\|\vec{r}\|n''p\rangle\langle n''p\|\vec{r}\|n's\rangle$$

$$\times\left\{ -\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & \underset{\uparrow\, m_\ell''}{0} \end{pmatrix}^2 e_0 e_0' + \begin{pmatrix} 0 & 1 & 1 \\ 0 & -1 & \underset{\uparrow\, m_\ell''}{1} \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ \underset{\uparrow\, -m_\ell''}{-1} & 1 & 0 \end{pmatrix} e_{+1} e'_{-1} \right.$$

$$\left. + \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix} e_{-1} e'_{+} \right\}$$

since $\langle j'm'|T_k^q|jm\rangle = (-1)^{j'-m'}\begin{pmatrix} j' & k & j \\ -m' & q & m \end{pmatrix}$

$$\times\langle j'\|T_k\|j\rangle$$

$$= (\hat{e}\cdot\hat{e}')\langle ns|z|np,0\rangle\langle np,0|z|n's\rangle$$

$\cancel{\sigma_{tot} = \sigma + \sigma'}$

If $\omega \neq 0$, final states are orthogonal and there is no interference. Cross-sections are additive $\sigma_{tot} = \sigma + \sigma'$.

$\cancel{\sigma_{tot}\, d\Omega_k = \omega\omega'}$

$\omega = 0$ case. - Static Field Quenching.

- Final states are identical and amplitudes are added.

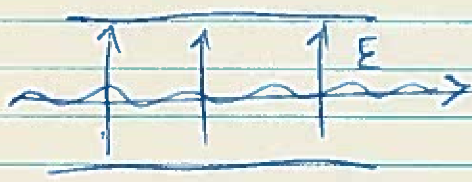
$$Q_{nn'}^{(tot)} = Q_{nn'} + Q'_{nn'}$$

But $Q_{nn'}(\omega=0) = Q'_{nn'}(\omega = 0)$

$$= \sum_{n''} \langle n|z|n''\rangle\langle n''|z|n'\rangle \times \left( \frac{1}{E_n - E_{n''}} + \frac{1}{E_{n'} - E_{n''}} \right)$$

$$\sigma_{tot}\, d\Omega_k = \frac{\omega\omega'^3}{c^4}\left(a_\perp^2 + a_\parallel^2\cos^2\theta_k\right) 4\,|Q_{nn'}(0)|^2\, d\Omega_k$$

$$\sigma_{tot} = \frac{8\pi}{3} \cdot \frac{4\omega\omega'^3}{c^4}\,|Q_{nn'}(0)|^2$$




very low frequency photons.

, $O(c^{-2})$ for E1.

Since $E = \frac{1}{8\pi}\dfrac{\int (\mathcal{E}^2 + \mathcal{H}^2)\, d\tau}{\Omega}$ = energy per unit vol.

$\mathcal{E}^2 = 8\pi E = 8\pi \hbar \frac{N\omega}{\Omega}$, $N$ = no. of photons. ~~per unit vol~~.

$\sigma = \dfrac{\text{no. scattered/unit } t}{I}$, $I = \dfrac{N}{\Omega} c$.

$\therefore$ no. scattered per unit time $= I\sigma = \frac{N}{\Omega} c\sigma$

$$= \left(8\pi \frac{N}{\Omega}\omega\right) \cdot \frac{4}{3}\frac{\omega'^3}{c^3} \left| Q_{nn'}(0) \right|^2$$

$$\therefore \mathcal{E}^2 \cdot \frac{4}{3}\frac{\omega'^3}{c^3} \left| Q_{nn'}(0) \right|^2 .$$

Can also be obtained by ordinary stationary state time-independent perturbation theory

eg He ($2^1S$)

$$|2^1S\rangle_1 = |2^1S\rangle + \mathcal{E}\sum_n \frac{|n^1P\rangle\langle n^1P|z|2^1S\rangle}{E(2^1S) - E(n^1P)}$$

$$|1^1S\rangle_1 = |1^1S\rangle + \mathcal{E}\sum_n \frac{|n^1P\rangle\langle n^1P|z|1^1S\rangle}{E(1^1S) - E(n^1P)}$$

$$A = \frac{4}{3}\frac{\omega'^3}{c^3} \left| \langle 2^1S|\underset{\sim}{r}|1^1S\rangle \right|^2$$

$$= \frac{4}{3}\frac{\omega'^3}{c^3} \left| {}_1\langle 2^1S|\underset{\sim}{r}|1^1S\rangle_0 + {}_0\langle 2^1S|\underset{\sim}{r}|1^1S\rangle_1 \right|^2$$

$$= \mathcal{E}^2 \cdot \frac{4}{3}\frac{\omega'^3}{c^3} \left| Q_{nn'}(0) \right|^2 = \tau^{-1}.$$

$A(\text{exp}) = (0.926 \pm 0.02)\, F^2\, (\text{cm}/\text{KeV})^2\, \text{sec}^{-1}$

(Petrasso & Ramsey, P.R. A 5, 79 (1972))

$A(\text{theory}) = (0.932 \pm 0.001)\, F^2\, \text{sec}^{-1}$.

(Drake.)

## Spontaneous Two-Photon Emission.

- Consider again stimulated two-photon emission.

$$P(n \to n') = \frac{(2\pi)^3 \omega\omega'}{\hbar^2} \frac{N_\lambda N_{\lambda'}}{\Omega^2} \left(a_\perp^2 + a_\parallel^2 \cos^2\theta_k\right)$$

$$\times \left| Q'_{nn'} \right|^2 \cdot \delta\left( E_n - E_{n'} - \hbar(\omega + \omega') \right)$$

- If both photons are emitted spontaneously, we multiply by final state density for both photons and integrate over all angles and directions of polarization.

$$\rho(\omega)\, d\Omega_k\, d\omega = \frac{\Omega \omega^2 d\omega\, d\Omega_k}{(2\pi)^3 c^3}$$

$$\rho(\omega')\, d\omega' = \Omega \omega^2 d\omega / (\pi^2 c^3)$$

$$A_{n \to n'} = \int P(n \to n') \rho(\omega) \rho(\omega')\, d\Omega_k\, d\Omega_{k'}\, d\omega\, d\omega'$$

with $N_\lambda = N_{\lambda'} = 1$,

$$A_{n \to n'} d\omega = (2\pi)^3 \times \left( \frac{1}{(2\pi)^3} \cdot \underbrace{\frac{8\pi}{3}}_{d\Omega_k} \right) \cdot \left( \underbrace{\frac{8\pi}{(2\pi)^3}}_{d\Omega_{k'}} \right)$$

$$\times \frac{\omega^3 \omega'^3}{\hbar^2 c^6} \left| Q'_{nn'} \right|^2 d\omega$$

$$= \frac{16}{3} \frac{\omega^3 \omega'^3}{\hbar^2 c^6} \left| Q'_{nn'} \right|^2 \frac{d\omega}{2\pi}$$

or $$A(\nu)\, d\nu = \frac{2^{10} \pi^6}{3 \hbar^2 c^6} \nu^3 \nu'^3 \left| Q'_{nn'} \right|^2 d\nu.$$

$$A_{n \to n'} = \frac{1}{2} \int_0^{\nu_{max}} A(\nu)\, d\nu.$$

$\nu_{max} = (E_n - E_{n'})/h.$

The event $(\nu, \nu')$ is indistinguishable from $(\nu', \nu)$ and so the factor of $1/2$ appears. i.e. only pairs of photons are counted.

Directional correlation factor $= 1 + \cos^2\theta_{12}$
Polarization correlation factor $= |\hat{e}_q \cdot \hat{e}_{q'}|^2$.

$\rightarrow$ particular to $J=0 \rightarrow J=1 \rightarrow J=0$.

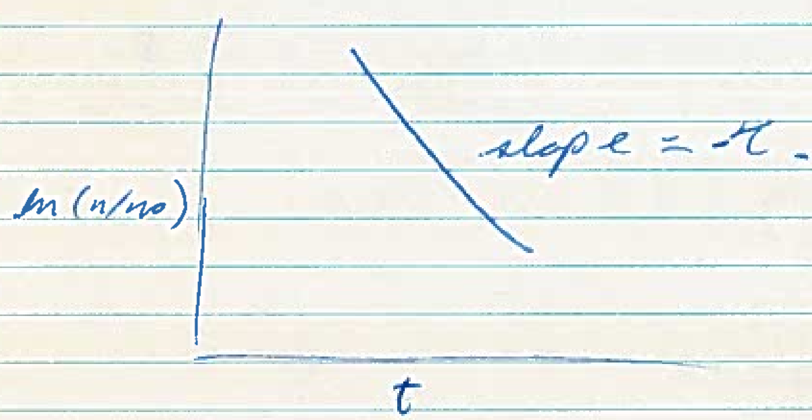
For He $2^1S$, $\tau$(theory) = 19.6 msec.

$\tau$(van Dyck, Johnson + Shugart) = 19.7±1 msec
Phys. Rev. A, 4, 1327 (1971)

$\tau$(Pearl) = 38 ± 8 msec.

$$n = n_0 e^{-t/\tau}$$

$$\ln(n/n_0) = -t/\tau.$$




Raman scattering and stimulated two-photon emission have been detected in H(2s) by Bräunlich and Lambropoulos, P.R.L. 25, 135 and 25, 986 (1971).